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New physics in $e^+e^- \to Z\gamma$ with polarized beams

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Abstract: We present a complete description of angular distributions in the presence of new interactions for the process $e^+e^- \to Z\gamma$ with polarized beams at future linear colliders, by considering the most general form-factors allowed by gauge invariance. We include the possibility of CP violation, and classify the couplings according to their CP properties. Chirality conserving and chirality violating couplings give rise to distinct dependence on beam polarization. We present a comprehensive discussion including both types of couplings and provide a detailed comparison of the effects due to each. We discuss some selected asymmetries which would enable isolating effects of the CP-violating form-factors. We also present sensitivities on the corresponding couplings achievable at a future linear collider with realistic polarization and luminosity.

Keywords: Beyond Standard Model, CP violation.

Contents

1. Introduction

An international linear collider (ILC) that will collide e^+ and e^- at centre of mass energy of $\sqrt{s} = 500$ GeV is now a distinct possibility. The aim of this machine would be to determine the parameters of the standard model (SM) at higher precision than ever before, discover the Higgs boson and establish its properties, produce particles that have so far not been accessible at present day energies, and probe physics even due to interactions mediated by particles that are too massive to be produced. One important window for the observation of beyond the standard model (BSM) physics is to establish CP violation outside the neutral meson system. As a result, it is important to discuss all the physics possibilities that would lead to such CP violation, the imprint each type of interaction would leave on measurements in as model independent a manner as possible, and to advance new and improved tools to probe such interactions. Thus, any work, like ours, which discusses the impact of new interactions should lay special emphasis on CP-violating observables.

It is by now clear that the availability of polarization of the beams, both longitudinal and transverse, would play an indisputable role in enhancing the sensitivity of observables to CP violation, and indeed would play complementary roles [1]. For instance, the availability of longitudinal polarization would enhance the possibility of detecting possible dipole moments of, say the τ-lepton [2] and the top-quark [3]. For the important process of $e^+e^- \to t\bar{t}$, the availability of transverse polarization would allow one to probe BSM scalar and tensor type interactions [4]. The last result was explicitly demonstrated some time ago, and could have been, in principle, deduced from the distinguished work of Dass and Ross [5]. The considerations here arise from properties of the interference of standard model amplitudes, which at tree-level are generated by the exchange of Z and γ in the $s-$channel with amplitudes represented by contact interactions due to BSM interactions. We note here that for light quarks, a discussion was presented in earlier work [6].

On the other hand, $Z\gamma$ production, a process for which there is a significant SM cross-section (for early work, see e.g. [7]), presents surprises since the SM contribution arises from the $t-$ and $u-$channel contributions, rather than $s-$channel contributions as in the top-quark case. In a recent article [8], we discussed at length the contributions to the differential cross-section due to completely model independent, most general gauge and Lorentz invariant, chirality conserving (CC) contact interactions. [Contact interactions have also been considered in the context of $s-$channel processes in, e.g., ref. [9 − 11].] In particular, one of these contact interactions generates precisely the same contributions as those generated by anomalous CP violating triple-gauge boson vertices studied in a similar context somewhat earlier [12, 13], when the parameters are suitably identified. In this paper, we complement the work of [8] by including a discussion of chirality violating (CV) contact interactions for new physics. The discussion here is thus comprehensive and includes the sum total of all Lorentz and gauge invariant interactions beyond the standard model. It thus provides a platform for a model independent discovery of BSM physics.

The term contact interactions, as we use it here, calls for some explanation. The term is usually used in a low-energy effective theory with a cut-off energy scale Λ for effective interactions induced in the form of nonrenormalizable terms by new physics at some high scale, and this consists of an expansion in inverse powers of Λ. The expansion is then terminated at some suitable inverse power of Λ, keeping effective higher-dimensional operators arising from new interactions up to a certain maximum dimension. In our approach we do not introduce a cut-off, nor do we limit the dimensions of the operators. We simply write down all independent forms of amplitudes in momentum space relevant for the process in question, with coefficients which are Lorentz invariant form-factors, and are functions of the kinematic invariants. Thus, in principle, we keep all powers of momenta, unlike in the low-energy effective theory approach.

Our formalism thus encompasses not only the standard contact interactions, but, for example, also interactions where there may be propagators for the exchange of new or SM virtual particles in the s, t or u channel.

We thus use the term contact interactions for convenience, to denote a general form-factor approach, in the spirit of Abraham and Lampe [14], in momentum space for the amplitude for the process in question (for a recent mention see [15]). It is worthwhile emphasizing that in the present framework, Λ does not play the role of a cut-off; therefore, the scale Λ will make no explicit appearance in our considerations. As a result, we will scale all BSM dimensionful parameters by the only mass scale in the problem which is m_Z, while noting that the latter only plays the role of a book-keeping device. To avoid confusion, we should state that even though we do not introduce explicitly a large energy scale Λ, our form-factors need not necessarily be of order unity — they may be suppressed by couplings constants and/or inverse powers of some heavy masses.

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In the absence of general results *à la* Dass and Ross for interactions of this type, it is not possible to guess what would happen if there were model independent form factor representation for BSM physics due to CV interactions. These could, in principle, be generated by either scalar type interactions containing no Dirac γ matrices or a γ_5 matrix, or tensor type interactions containing an anti-symmetrized product of two γ matrices, $\sigma_{\mu\nu}$. It turns out that in the limit of vanishing electron mass, the contributions of the latter always reduce to certain combinations indistinguishable from those induced by scalar type interactions. While our explicit computations realize this, which we do not report here, it may be seen to follow from some general considerations which we prove. Therefore, the linearly independent set of form-factors we employ here are those that are of the scalar type.

After presenting the results from the scalar form factors, we construct some sample asymmetries and evaluate them in terms of the new interactions. This enables us to provide examples of limits on the sensitivities that can be achieved in future collider experiments. We then discuss models in which such interactions can be generated. In addition to presenting our novel results, here we also present a comprehensive discussion on all the issues involved, including recounting important aspects of established results.

We begin with a general discussion on chirality conservation and violation in general and the role of transverse polarization in uncovering interactions of each type in section 2. This discussion parallels the one presented by Hikasa [16], which we shall briefly outline. We then specialize to the process of interest in section 3. We present a detailed discussion and summarize our conclusions in section 4. We provide proof of the redundancy of tensor form-factors in appendix A, while appendix B contains a discussion of the CP properties of various couplings in the contact interactions.

2. Chirality conservation and violation

Polarization effects are different for new interactions which are chirality conserving and for those which are chirality violating. Firstly, in the limit of vanishing electron mass, there is no interference of the chirality violating new interactions with the the SM interactions, since the latter are chirality conserving. As a result, there is no contribution from chirality violating interactions which is polarization independent or dependent on longitudinal polarization in the limit of vanishing electron mass. Transverse polarization effects for the two cases are also different. The cross terms of the SM amplitude with the amplitude from chirality conserving contact interactions has a part independent of transverse polarization and a part which is bilinear in transverse polarization of the electron and positron, denoted by P_T and $\overline{P}_T$ respectively. For the case of chirality violating interactions, the cross term has only terms linear in P_T and $\overline{P}_T$, and no contributions independent of these.

The features discussed above have been captured in the work of Hikasa [16], wherein the cross-section Σ for the transversely polarized case is expressed as

$$\Sigma = \Sigma_{\rm unpol} - \frac{1}{2}P_T\overline{P}_T\,{\rm Re}[T^*_{++}T_{--}] + \frac{1}{2}P_T\overline{P}_T\,{\rm Re}[e^{-2i\phi_H}T^*_{+-}T_{-+}] + \\ +\frac{1}{2}P_T\,{\rm Re}[e^{-i\phi_H}\left(T^*_{+-}T_{--} + T^*_{++}T_{-+}\right)] - \frac{1}{2}\overline{P}_T\,{\rm Re}[e^{-i\phi_H}(T^*_{++}T_{-+} + T^*_{--}T_{-+})]\,,$$

where the T_{++}, T_{+-}, T_{-+} and T_{--} are helicity amplitudes for the process at hand, ϕ_H is the final-state azimuthal angle and $\Sigma_{\rm unpol}$ is the unpolarized cross-section. In the T_{ab}, $a, b = +, -$, the subscripts stand for the helicities of the e^+ and e^- respectively. In the above, BSM interactions of the CC type contribute to the amplitudes T_{+-} and T_{-+}, while those of the CV type contribute to $T_{aa}, a = +, -$ (the SM interactions themselves contribute only to T_{+-} and T_{-+}, when m_e effects are neglected). Note also the characteristic $2\phi_H$ dependence accompanying the terms bilinear in transverse polarization, and the ϕ_H dependence accompanying the linear transverse polarization pieces when the BSM physics is worked out to leading order. In this manner, the general treatment of Hikasa provides a useful reference guide to the polarization dependence of the cross-sections on BSM interactions, and can be used to check the observations of the preceding paragraph.

Due to the above-mentioned dependence on transverse polarization, the type of CP violating observables is also different in the two cases of chirality conservation and chirality violation. It is thus clear that the two cases should be treated separately, and this is what we do in the following. From hereon, we shall specialize to the process of interest in this work, *viz.* $Z\gamma$ production.

3. The process $e^+e^- \to Z\gamma$ with contact interactions

In this section we consider the process

$$e^-(p_-, s_-) + e^+(p_+, s_+) \to \gamma(k_1, \alpha) + Z(k_2, \beta)\,, \tag{3.1}$$

parametrize the contribution to its amplitude in terms of form-factors introduced in a contact-interaction description of physics beyond the standard model, and discuss asymmetries in the consequent angular distributions.

3.1 Contact interactions with chirality conservation and violation

We shall assume that the amplitudes are generated by the standard model as well as a general set of form-factors of the type proposed by Abraham and Lampe [14] (see also ref. [17]). They are completely determined by vertex factors that we denote by $\Gamma^{SM}_{\alpha\beta}$, $\Gamma^{CC}_{\alpha\beta}$ and $\Gamma^{CV}_{\alpha\beta}$ in a self-explanatory notation. Of these, $\Gamma^{CV}_{\alpha\beta}$ is being proposed here for the first time.

The vertex factor corresponding to SM is given by

$$\Gamma^{SM}_{\alpha\beta} = \frac{e^2}{4\sin\theta_W\cos\theta_W}\left\{\gamma_\beta(g_V - g_A\gamma_5)\frac{1}{\not p_- - \not k_1}\gamma_\alpha + \gamma_\alpha\frac{1}{\not p_- - \not k_2}\gamma_\beta(g_V - g_A\gamma_5)\right\}. \tag{3.2}$$

In the above, the vector and axial vector Z couplings of the electron are

$$g_V = -1 + 4\sin^2\theta_W\,; \qquad g_A = -1\,. \tag{3.3}$$

The chirality conserving anomalous form factors may be introduced via the following vertex factor, which is denoted here by:

$$\begin{aligned}\Gamma^{CC}_{\alpha\beta} = \frac{ie^2}{4\sin\theta_W\cos\theta_W}\Bigg\{ & \frac{1}{m_Z^4}\Big((v_1+a_1\gamma_5)\gamma_\beta(2p_{-\alpha}(p_+\cdot k_1)-2p_{+\alpha}(p_-\cdot k_1))+\\ & +((v_2+a_2\gamma_5)p_{-\beta}+(v_3+a_3\gamma_5)p_{+\beta})(\gamma_\alpha 2p_-\cdot k_1-2p_{-\alpha}\not{k}_1)+\\ & +((v_4+a_4\gamma_5)p_{-\beta}+(v_5+a_5\gamma_5)p_{+\beta})(\gamma_\alpha 2p_+\cdot k_1-2p_{+\alpha}\not{k}_1)\Big)+\\ & +\frac{1}{m_Z^2}(v_6+a_6\gamma_5)(\gamma_\alpha k_{1\beta}-\not{k}_1 g_{\alpha\beta})\Bigg\}. \end{aligned} \tag{3.4}$$

This was discussed by us earlier in [8].

We now introduce the corresponding CV form-factors. In terms of a linearly independent set of scalar form-factors (i.e., ones with no Dirac γ's), the vertex factor can be written down as:

$$\begin{aligned}\Gamma^{CV}_{\alpha\beta} = & \frac{e^2}{4\sin\theta_W\cos\theta_W}\cdot\\ & \cdot\Bigg((s_1+ip_1\gamma_5)\frac{[(k_1\cdot k_2)g_{\alpha\beta}-k_{1\beta}k_{2\alpha}]}{m_Z^3}+\\ & +(s_2+ip_+\gamma_5)p_{-\beta}\frac{((k_1\cdot p_+)p_{-\alpha}-(k_1\cdot p_-)p_{+\alpha})}{m_Z^5}+\\ & +(s_3+ip_3\gamma_5)p_{+\beta}\frac{((k_1\cdot p_+)p_{-\alpha}-(k_1\cdot p_-)p_{+\alpha})}{m_Z^5}+\frac{(s_4+ip_4\gamma_5)\epsilon_{\alpha\beta\rho\sigma}k_1^\rho k_2^\sigma}{m_Z^3}+\\ & +(s_5+ip_5\gamma_5)\epsilon_{\alpha\beta\rho\sigma}k_1^\rho\frac{(p_--p_+)^\sigma}{m_Z^3}+(s_6+ip_6\gamma_5)\times\\ & \times\frac{[(2(k_1\cdot p_-)(k_1\cdot p_+)\epsilon_{\alpha\beta\sigma\tau}+\epsilon_{\beta\rho\sigma\tau}k_1^\rho(p_{-\alpha}(k_1\cdot p_+)+p_{+\alpha}(k_1\cdot p_-)))\,p_-^\sigma p_+^\tau]}{m_Z^7}\Bigg). \end{aligned} \tag{3.5}$$

The form-factors introduced above are in principle functions of the Lorentz invariant quantities s and t. In a specific frame (as for example the e^+e^- centre-of-mass frame which we employ) they could be written as functions of s and $\cos\theta$, where θ is the production angle of γ. However, in what follows, we will assume for simplicity that the form-factors are all constants. To the extent that we restrict ourselves to a definite e^+e^- centre-of-mass energy, the absence of s dependence is not a strong assumption. However, the absence of θ dependence is a strong assumption, and relaxing this assumption can have important consequences, as discussed below. We begin by recalling that one can write form-factors, which are functions of the Mandelstam variables s, t, u as functions of just s and $t-u$, since

$$s+t+u=m_Z^2\,. \tag{3.6}$$

Further recalling that $t-u=(s-m_Z^2)\cos\theta$, our form-factors are, in general, functions $F_i(s,\cos\theta)$ of s and $\cos\theta$. Furthermore, we can write each form-factor as a sum of even and odd parts as

$$F_i(s,\cos\theta)=f_i(s,\cos\theta)+g_i(s,\cos\theta)\,, \tag{3.7}$$

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CP even	CP odd
v_1	
$v_2 - v_5$	$v_2 + v_5$
$v_3 - v_4$	$v_3 + v_5$
	v_6
a_1	
$a_2 - a_5$	$a_2 + a_5$
$a_3 - a_4$	$a_3 + a_5$
	a_6

Table 1: CP even and odd combinations of CC couplings.

CP even	CP odd
s_1	
$s_2 - s_3$	$s_2 + s_3$
	s_4
s_5	
s_6	
	p_1
$p_2 + p_3$	$p_2 - p_3$
p_4	
	p_5
	p_6

Table 2: CP even and odd combinations of CV couplings.

where the f_i are even and g_i odd functions of $\cos\theta$. Note that $\cos\theta$ changes sign under CP. Hence, if F_i occurs with a certain tensor which is CP conserving (violating), then the f_i part contributes an amount which is CP conserving (violating), and the g_i part an amount which is CP violating (conserving). In principle, our analysis can be done taking the f_i and g_i in to account. However, it would be extremely complicated given the large number of form factors.

As noted in [8], the combinations $v_1, v_2 - v_5, v_3 - v_4$ and $a_1, a_2 - a_5, a_3 - a_4$ are CP conserving, while $v_2 + v_5, v_3 + v_4, v_6$ and $a_2 + a_5, a_3 + a_4, a_6$ and are CP violating, assuming that they are functions only of s. As for the form-factors for the CV interactions, the combinations $s_1, s_2 - s_3, s_5, s_6, p_2 + p_3$ and p_4 are CP conserving and $s_2 + s_3, s_4, p_1, p_2 - p_3, p_5$ and p_6 are CP violating, again assuming them to be functions of s alone. (In tables 1 and 2 we present the CP properties of all the combinations of couplings of interest.) In appendix B we demonstrate how the CP properties of the couplings may be determined and derive the consequences of the CPT theorem.

A discussion is in order on the number of CP violating form-factors we expect to have. Given that the Z is a massive vector particle and that the photon is a massless one, and that the electron is a spin 1/2 particle, we have 12 helicity amplitudes. We can, therefore, have 12 form-factors in the chirality conserving case, neglecting the electron mass. Of the v_i, $a_i, i = 1, \ldots, 6$, only three linear combinations of each are CP violating, and since each is complex, the total number is 12 as expected, and so also for the CP conserving case. The count is analogous also for the chirality violating case considered here with s_i, $p_i, i = 1, \ldots, 6$.

3.2 The differential cross-section

We now give expressions for the differential cross-sections including both CC and CV contact interactions for polarized beams. The differential cross-section for longitudinal beam polarizations P_L and $\overline{P}_L$ of e^- and e^+ is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_L = \mathcal{B}_L \left(1 - P_L\overline{P}_L\right) \left[\frac{1}{\sin^2\theta}\left(1 + \cos^2\theta + \frac{4\overline{s}}{(\overline{s}-1)^2}\right) + C_L\right], \tag{3.8}$$

i	X_i	Y_i	Z_i
1	$-2\overline{s}(\overline{s}+1)$	0	$-4\sqrt{\overline{s}}\cot\theta$
2	$\overline{s}(\overline{s}-1)(\cos\theta-1)$	0	$-\overline{s}^{3/2}(\overline{s}-1)\sin\theta/2$
3	0	$\overline{s}(\overline{s}-1)(\cos\theta-1)$	$-\overline{s}^{3/2}(\overline{s}-1)\sin\theta/2$
4	0	$\overline{s}(\overline{s}-1)(\cos\theta+1)$	$4\sqrt{\overline{s}}\csc\theta$
5	$\overline{s}(\overline{s}-1)(\cos\theta+1)$	0	$-4\sqrt{\overline{s}}\cot\theta$
6	$2(\overline{s}-1)\cos\theta$	$2(\overline{s}-1)\cos\theta$	$-\overline{s}^{3/2}(\overline{s}-1)^2\sin 2\theta/8$

Table 3: The contribution of the new couplings to the cross section.

where

$$\overline{s}\equiv\frac{s}{m_Z^2}\,,\qquad \mathcal{B}_L=\frac{\alpha^2}{16\sin^2\theta_W m_W^2\overline{s}}\left(1-\frac{1}{\overline{s}}\right)(g_V^2+g_A^2-2Pg_Vg_A)\,, \tag{3.9}$$

where the effective polarization parameter P is defined as

$$P=\frac{P_L-\overline{P}_L}{1-P_L\overline{P}_L}\,, \tag{3.10}$$

and

$$C_L=\frac{1}{4(g_V^2+g_A^2-2Pg_Vg_A)}\left\{\sum_{i=1}^{6}\left((g_V-Pg_A)\operatorname{Im}v_i+(g_A-Pg_V)\operatorname{Im}a_i\right)X_i\right\}, \tag{3.11}$$

and where the $X_i, i=1,\ldots,6$ are kinematical quantities resulting from our computations and are listed in table 3 (along with $Y_i, i=1,\ldots,6$ and $Z_i, i=1,\ldots,6$ which enter transversely polarized cross-sections in the following). Note that our convention is one in which positive P_L corresponds to right polarized electrons and positive $\overline{P}_L$ to right polarized positrons. The differential cross-section for transverse polarizations P_T and $\overline{P}_T$ of e^- and e^+ is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_T=\mathcal{B}_T\left[\frac{1}{\sin^2\theta}\left(1+\cos^2\theta+\frac{4\overline{s}}{(\overline{s}-1)^2}-P_T\overline{P}_T\frac{g_V^2-g_A^2}{g_V^2+g_A^2}\sin^2\theta\cos 2\phi\right)+C_T^{CC}+C_T^{CV}\right], \tag{3.12}$$

with,

$$\mathcal{B}_T=\frac{\alpha^2}{16\sin^2\theta_W m_W^2\overline{s}}\left(1-\frac{1}{\overline{s}}\right)(g_V^2+g_A^2)\,, \tag{3.13}$$

and

$$\begin{aligned}C_T^{CC}=\frac{1}{4(g_V^2+g_A^2)}\Bigg\{&\sum_{i=1}^{6}(g_V\operatorname{Im}v_i+g_A\operatorname{Im}a_i)X_i+\\&+P_T\overline{P}_T\sum_{i=1}^{6}\Big((g_V\operatorname{Im}v_i-g_A\operatorname{Im}a_i)\cos 2\phi+\\&+(g_A\operatorname{Re}v_i-g_V\operatorname{Re}a_i)\sin 2\phi\Big)Y_i\Bigg\}\,,\end{aligned} \tag{3.14}$$

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and

$$\begin{aligned}
C_T^{CV} &= \frac{1}{8(g_V^2+g_A^2)}\cdot \\
&\cdot\Bigg(\sum_{i=1}^{3}\Big\{(P_T-\overline{P}_T)[g_V(\text{Im } s_i)\sin\phi - g_A(\text{Re } s_i)\cos\phi]+ \\
&\qquad\qquad + (P_T+\overline{P}_T)[g_A(\text{Re } p_i)\sin\phi + g_V(\text{Im } p_i)\cos\phi]\Big\}\cdot Z_i + \\
&\quad + \sum_{i=4}^{6}\Big\{(P_T-\overline{P}_T)[g_V(\text{Im } s_i)\cos\phi + g_A(\text{Re } s_i)\sin\phi]+ \\
&\qquad\qquad + (P_T+\overline{P}_T)[g_A(\text{Re } p_i)\cos\phi - g_V(\text{Im } p_i)\sin\phi]\Big\}\cdot Z_i\Bigg). \qquad (3.15)
\end{aligned}$$

As expected, there is no contribution from the CV interactions to the cross-section (3.8) with longitudinal polarization.

In the expressions above, θ is the angle between photon and the e^- directions, and ϕ is the azimuthal angle of the photon, with the e^- direction chosen as the z axis, and with the direction of its transverse polarization chosen as the x axis. The e^+ transverse polarization direction is chosen anti-parallel to the e^- transverse polarization direction[1], and corresponds to the convention adopted by Hikasa [16].

We have kept only terms of leading order in the anomalous couplings, since they are expected to be small. The above expressions may be obtained either by using standard trace techniques for Dirac spinors with a transverse spin four-vector, or by first calculating helicity amplitudes and then writing transverse polarization states in terms of helicity states. We note that the contribution of the interference between the SM amplitude and the anomalous amplitude vanishes for $s = m_Z^2$. The reason is that for $s = m_Z^2$ the photon in the final state is produced with zero energy and momentum, and for the photon four-momentum $k_1 = 0$, the anomalous contribution (3.4) vanishes identically.

3.3 Asymmetries

We now formulate angular asymmetries which can help to determine different independent linear combinations of form-factors. The number of form-factors in either the CC case or the CV case is 12. A glance at table 3 reveals that the number of independent angular distributions in either case is not that large. Thus, even if electron and positron polarizations can be turned on or off, or allowed to change signs, it would not be possible to determine all the form-factors from an experimental determination of the polarized angular distributions.

We discuss below some selected asymmetries which can be useful. Our choice of asymmetries would be ideally suited to determine the CP-violating combinations of form-factors in the case when one could choose electron and positron polarizations to be equal in magnitude but opposite in sign, so that the initial state has a definite CP transformation. In

[1] This was incorrectly stated as “parallel” in [8, 13].

such a case, it may be checked using the CP properties of form-factors detailed in section (2.1) that the form-factors appearing in the asymmetries chosen below are precisely in the combinations which are odd under CP. In all cases, we use a cut-off θ_0 in the forward and backward directions on the polar angle θ of the photon. This cut-off is needed since no observation can be made too close to the beam direction. Moreover, it can serve a further purpose that the sensitivity can be optimized by choosing a suitable cut-off.

For the case of the CC, we introduce the following asymmetries [8]:

$$A_1^{CC}(\theta_0) = \frac{1}{\sigma_0}\sum_{n=0}^{3}(-1)^n\left(\int_0^{\cos\theta_0} d\cos\theta - \int_{-\cos\theta_0}^{0} d\cos\theta\right)\int_{\pi n/2}^{\pi(n+1)/2} d\phi\,\frac{d\sigma}{d\Omega}, \tag{3.16}$$

$$A_2^{CC}(\theta_0) = \frac{1}{\sigma_0}\sum_{n=0}^{3}(-1)^n\left(\int_0^{\cos\theta_0} d\cos\theta - \int_{-\cos\theta_0}^{0} d\cos\theta\right)\int_{\pi(2n-1)/4}^{\pi(2n+1)/4} d\phi\,\frac{d\sigma}{d\Omega}, \tag{3.17}$$

and

$$A_3^{CC}(\theta_0) = \frac{1}{\sigma_0}\left(\int_{-\cos\theta_0}^{0} d\cos\theta - \int_0^{\cos\theta_0} d\cos\theta\right)\int_0^{2\pi} d\phi\,\frac{d\sigma}{d\Omega}, \tag{3.18}$$

with

$$\sigma_0 \equiv \sigma_0(\theta_0) = \int_{-\cos\theta_0}^{\cos\theta_0} d\cos\theta\int_0^{2\pi} d\phi\,\frac{d\sigma}{d\Omega}. \tag{3.19}$$

Of the asymmetries above, A_1^{CC} and A_2^{CC} exist only in the presence of transverse polarization. The asymmetry A_3^{CC}, on the other hand is enhanced in the presence of longitudinal polarization, compared to when the beams are unpolarized. These are readily evaluated and read as follows:

$$\begin{aligned} A_1^{CC}(\theta_0) &= \mathcal{B}_T'\,P_T\overline{P}_T\Big[g_A\left\{\overline{s}(\mathrm{Re}\,v_3 + \mathrm{Re}\,v_4) + 2\,\mathrm{Re}\,v_6\right\} - \\ &\qquad - g_V\left\{\overline{s}(\mathrm{Re}\,a_3 + \mathrm{Re}\,a_4) + 2\,\mathrm{Re}\,a_6\right\}\Big], \end{aligned} \tag{3.20}$$

$$\begin{aligned} A_2^{CC}(\theta_0) &= \mathcal{B}_T'\,P_T\overline{P}_T\Big[g_V\left\{\overline{s}(\mathrm{Im}\,v_3 + \mathrm{Im}\,v_4) + 2\,\mathrm{Im}\,v_6\right\} - \\ &\qquad - g_A\left\{\overline{s}(\mathrm{Im}\,a_3 + \mathrm{Im}\,a_4) + 2\,\mathrm{Im}\,a_6\right\}\Big], \end{aligned} \tag{3.21}$$

where, we have defined

$$\mathcal{B}_T' = \frac{\mathcal{B}_T(\overline{s}-1)\cos^2\theta_0}{(g_V^2+g_A^2)\sigma_0^T}, \tag{3.22}$$

with

$$\sigma_0^T = 4\pi\mathcal{B}_T\left[\left\{\frac{\overline{s}^2+1}{(\overline{s}-1)^2}\ln\left(\frac{1+\cos\theta_0}{1-\cos\theta_0}\right) - \cos\theta_0\right\}\right]. \tag{3.23}$$

We note here that A_1^{CC} (A_2^{CC}) are sensitive to real (imaginary) parts of v_3, a_3, v_4, a_4, v_6 and v_6. Indeed, as mentioned in earlier, these asymmetries bring in the transverse e^+ and e^- polarizations bilinearly in accordance with the general expectations presented in section 2.

The asymmetry A_3 which is independent of transverse polarization is found to be

$$\begin{aligned} A_3^{CC}(\theta_0) &= \mathcal{B}_L'\frac{\pi}{2}\Big[(g_A - Pg_V)\left\{\overline{s}(\mathrm{Im}\,a_2 + \mathrm{Im}\,a_5) + 2\,\mathrm{Im}\,a_6\right\} + \\ &\qquad + (g_V - Pg_A)\left\{\overline{s}(\mathrm{Im}\,v_2 + \mathrm{Im}\,v_5) + 2\,\mathrm{Im}\,v_6\right\}\Big], \end{aligned} \tag{3.24}$$

where

$$\mathcal{B}'_L = \frac{\mathcal{B}_L(1-P_L\overline{P}_L)(\overline{s}-1)\cos^2\theta_0}{(g_V^2+g_A^2-2Pg_Vg_A)\sigma_0^L}, \tag{3.25}$$

and

$$\sigma_0^L = 4\pi\mathcal{B}_L(1-P_L\overline{P}_L)\left[\left\{\frac{\overline{s}^2+1}{(\overline{s}-1)^2}\ln\left(\frac{1+\cos\theta_0}{1-\cos\theta_0}\right)-\cos\theta_0\right\}\right]. \tag{3.26}$$

In accordance with the CPT theorem A_3^{CC} is sensitive only to imaginary parts of the couplings v_2, a_2, v_5, a_5, v_6 and a_6.

For the case of CV we have chosen two types of forward-backward asymmetries, which happen to involve only 6 out of the 12 form-factors. We define the following asymmetries:

$$A_1^{CV}(\theta_0) = \frac{1}{\sigma_0^T}\sum_{i=0}^{1}(-1)^n\left(\int_{-\cos\theta_0}^{0}d\cos\theta-\int_0^{\cos\theta_0}d\cos\theta\right)\int_{n\pi}^{(n+1)\pi}d\phi\,\frac{d\sigma}{d\Omega}, \tag{3.27}$$

$$A_2^{CV}(\theta_0) = \frac{1}{\sigma_0^T}\sum_{i=0}^{1}(-1)^n\left(\int_{-\cos\theta_0}^{0}d\cos\theta-\int_0^{\cos\theta_0}d\cos\theta\right)\int_{(n-1/2)\pi}^{(n+1/2)\pi}d\phi\,\frac{d\sigma}{d\Omega} \tag{3.28}$$

which may be evaluated to read:

$$\begin{aligned}A_1^{CV}(\theta_0) = {} & \frac{\mathcal{B}_T}{12(g_V^2+g_A^2)\sigma_0^T}\cdot \\ & \cdot\Big((P_T-\overline{P}_T)g_A(48\,\mathrm{Re}\,s_5+\mathrm{Re}\,s_6(\sqrt{\overline{s}}-1)^2\overline{s}(1+\sin^2\theta_0+\sin\theta_0))+ \\ & \quad+(P_T+\overline{P}_T)g_V(48\,\mathrm{Im}\,p_5+\mathrm{Im}\,p_6(\sqrt{\overline{s}}-1)^2\overline{s}(1+\sin^2\theta_0+\sin\theta_0))\Big)\times \\ & \times\overline{s}(\sin\theta_0-1)+\frac{4\mathcal{B}_T}{(g_V^2+g_A^2)\sigma_0^T}\times \\ & \times((P_T-\overline{P}_T)g_V\,\mathrm{Im}\,s_1+(P_T+\overline{P}_T)g_A\,\mathrm{Re}\,p_1)\sqrt{\overline{s}}(\sin\theta_0-1)\end{aligned} \tag{3.29}$$

and

$$\begin{aligned}A_2^{CV}(\theta_0) = {} & \frac{\mathcal{B}_T}{12(g_V^2+g_A^2)\sigma_0^T}\cdot \\ & \Big((P_T-\overline{P}_T)g_V(48\,\mathrm{Im}\,s_5+\mathrm{Im}\,s_6(\sqrt{\overline{s}}-1)^2\overline{s}(1+\sin^2\theta_0+\sin\theta_0))+ \\ & \quad+(P_T+\overline{P}_T)g_A(48\,\mathrm{Re}\,p_5+\mathrm{Re}\,p_6(\sqrt{\overline{s}}-1)^2\overline{s}(1+\sin^2\theta_0+\sin\theta_0))\Big)\times \\ & \times\overline{s}(\sin\theta_0-1)+\frac{4\mathcal{B}_T}{(g_V^2+g_A^2)\sigma_0^T}\times \\ & \times((P_T-\overline{P}_T)g_A\,\mathrm{Re}\,s_1+(P_T+\overline{P}_T)g_V\,\mathrm{Im}\,p_1)\sqrt{\overline{s}}(\sin\theta_0-1).\end{aligned} \tag{3.30}$$

The asymmetry A_1^{CV} is sensitive to the real parts of the couplings s_5, s_6 and p_1 and the imaginary parts of the couplings p_5, p_6 and s_1, while the asymmetry A_2^{CV} is sensitive to the imaginary parts of the couplings s_5, s_6 and p_1 and the real parts of the couplings p_5, p_6 and s_1. Indeed, these asymmetries bring in the transverse e^+ and e^- polarizations linearly in accordance with the general considerations of section 2.

In the following section, we employ these asymmetries to provide an estimate of sensitivities attainable at the linear collider with realistic degrees of polarization and integrated luminosity.

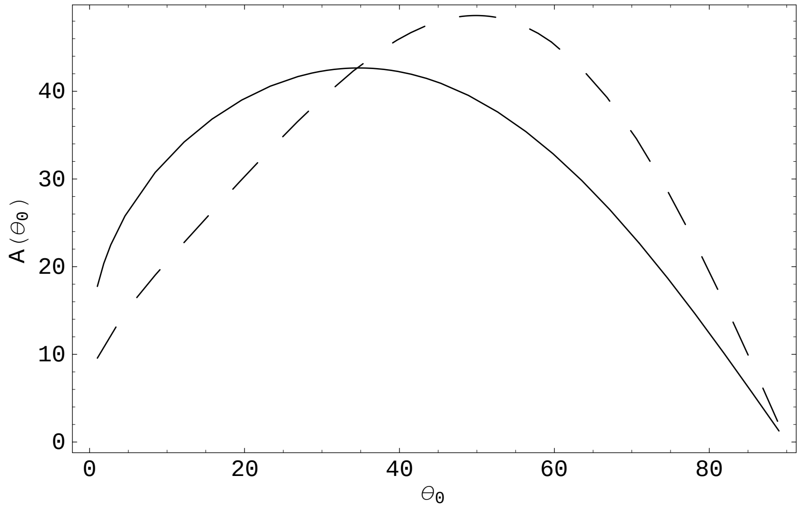

Figure 1: Value of asymmetry $A_1^{CV}(\theta_0)(\times 10^3)$ obtained with $\mathrm{Im}s_1 = 1$ (solid) and $A_2^{CV}(\theta_0)$ with $\mathrm{Im}s_6 = 1$ (dashed).

3.4 Sensitivities

In this section, we shall present a brief discussion on the sensitivity that can be obtained on the couplings at a linear collider with realistic polarization and luminosity. This is meant merely for the purposes of illustration, and we shall provide a simplified discussion, assuming only one coupling nonzero at a time. We assume realistic values of polarization and discuss the cases of same-sign and opposite-sign polarizations separately. This has the advantage of isolating, e.g., the scalar and pseudo-scalar couplings separately, etc.. Such a mode of operation is likely at the future linear collider. We shall separately present the sensitivities obtained for the CV and CC cases in the following.

3.4.1 Chirality violating couplings

In order to obtain the sensitivities of the CV couplings, we make use of the asymmetries A_i^{CV} $(i = 1, 2)$. We first consider the asymmetry A_1^{CV} and assume only non-vanishing imaginary parts. We also assume that $P_T = 0.8$, and $\overline{P}_T = -0.6$. For this simplified case, assuming $\mathrm{Im}s_1 = 1$, and the remaining $\mathrm{Im}s_i$ to be vanishing, we can compute the asymmetry, plotted as the solid profile in figure 1. The asymmetry A_2^{CV} is the same with the choice $\mathrm{Im}s_5 = 1$, while the computed asymmetry with the choice $\mathrm{Im}s_6 = 1$ is given by the dashed profile in figure 1.

We have here assumed $\sqrt{s} = 500\,\mathrm{GeV}$, $\int L dt = 500\,\mathrm{fb}^{-1}$, and magnitudes of electron and positron polarization to be 0.8 and 0.6 respectively. We have calculated 90% CL limits that can be obtained with a LC with the operating parameters given above. Denoting this sensitivity by the symbol δ (i.e., the respective real or imaginary part of the coupling), it is related to the value A of a generic asymmetry for unit value of the relevant coupling constant by:

$$\delta \equiv \frac{1.64}{|A|\sqrt{N_{SM}}}, \tag{3.31}$$

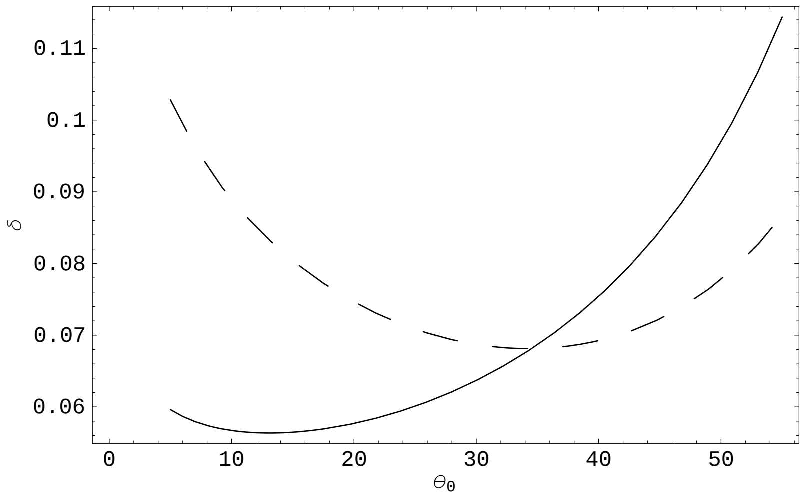

Figure 2: Value of sensitivities from A_1^{CV} on $\mathrm{Im} s_1$ (solid) and from A_2^{CV} on $\mathrm{Im} s_6 (\times 10^3)$ (dashed).

where N_{SM} is the number of SM events[2]. Our results are plotted in figure 2. We may now optimize the sensitivity at the following angles, giving us the following numbers: $|\,\mathrm{Im}\, s_{1,5}| \leq 5.6 \cdot 10^{-2}$, (optimum angle of 13^0), $|\,\mathrm{Im}\, s_6| \leq 6.8 \cdot 10^{-5}$ (optimum angle of 34^0). These may be readily translated into sensitivities for the other couplings: the sensitivity of $\mathrm{Re} s_{1,5,6}$ is that of the corresponding imaginary part multiplied by $g_V/g_A \simeq 0.08$ which yields: $|\,\mathrm{Re}\, s_{1,5}| \leq 4.5 \cdot 10^{-3}$, (optimum angle of 13^0), $|\,\mathrm{Re}\, s_6| \leq 5.4 \cdot 10^{-6}$ (optimum angle of 34^0). We keep in mind that these sensitivities are obtained by suitably interchanging the asymmetries $A_1^{CV} \leftrightarrow A_2^{CV}$, and switching the sign of the positron polarization. Finally, we note that the sensitivities of the real and imaginary parts of the $p_{1,5,6}$ is identical to those of their scalar counterparts.

3.4.2 Chirality conserving couplings

We now come to the asymmetries and sensitivities in the CC case, which were already reported in ref. [8]. We take up for illustration the case when only $\mathrm{Re}\, v_6$ is nonzero, since the results for other CP-violating combinations can be deduced from this case. We choose $P_T = 0.8$ and $\overline{P}_T = 0.6$, and vanishing longitudinal polarization for this case. Figure 3 shows the asymmetries A_i^{CC} as a function of the cut-off when the values of the anomalous couplings $\mathrm{Re}\, v_6$ (for the case of A_1^{CC}) and Im v_6 (for the case of A_2^{CC} and A_3^{CC}) alone are set to unity. We have again calculated 90% CL limits that can be obtained with a LC with $\sqrt{s} = 500\,\mathrm{GeV}$, $\int L dt = 500\,\mathrm{fb}^{-1}$, $P_T = 0.8$, and $\overline{P}_T = 0.6$ making use of the asymmetries A_i^{CC} $(i = 1, 2)$. For A_3^{CC}, we assume unpolarized beams. The curves from A_1^{CC} corresponding to setting only Re v_6 nonzero, and from A_2^{CC} and A_3^{CC} corresponding to keeping only Im v_6 nonzero are illustrated in figure 4. That there is a stable plateau for a choice of θ_0 such that $10° \lesssim \theta_0 \lesssim 40°$; and we choose the optimal value of 26^0 (we note here that the angle is the same for all cases considered, unlike in the CV case). The sensitivity

[2]The coefficient 1.64 may be obtained from statistical tables for hypothesis testing with one estimator, see, e.g., table 32.1 of ref. [18].

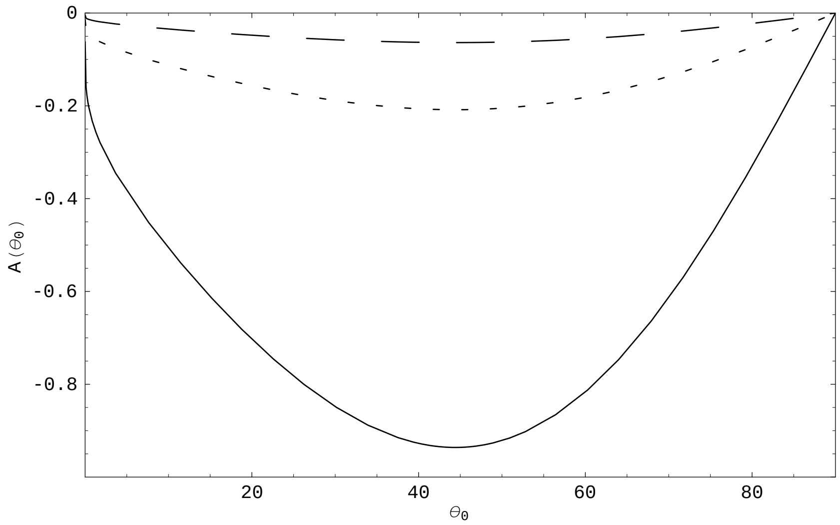

Figure 3: The asymmetries $A_1^{CC}(\theta_0)$ (solid line), $A_2^{CC}(\theta_0)$ (dashed line) and $A_3^{CC}(\theta_0)$ (dotted line), defined in the text, plotted as functions of the cut-off θ_0 for a value of Re $v_6 =$ Im $v_6 = 1$.

corresponding to this for Re v_6 is $\sim 3.1 \cdot 10^{-3}$. The results for the other couplings may be inferred in a straightforward manner from the explicit example above. For the asymmetry A_1^{CC}, if we were to set $v_3(v_4)$ to unity, with all the other couplings to zero, then the asymmetry would be simply scaled up by a value $\overline{s}/2$, which for the case at hand is $\simeq 14.8$. The corresponding limiting value would be suppressed by the reciprocal of this factor. The results for the couplings $\mathrm{Re}\, a_{2,5,6}$, compared to what we have for the vector couplings would be scaled by a factor g_V/g_A for the asymmetries and by the reciprocal of this factor for the sensitivities. The results coming out of the asymmetry A_2^{CC} are such that the sensitivities of the imaginary parts of v and a are interchanged *vis à vis* what we have for the real parts coming out of A_1^{CC}. The final set of results we have is for the form-factors that may be analyzed via the asymmetry A_3^{CC}, which depends only on longitudinal polarizations. We treat the cases of unpolarized beams and longitudinally polarized beams with $P_L = 0.8$, and $\overline{P}_L = -0.6$ separately. For the unpolarized case, the results here for Im v_6 correspond to those coming from A_2^{CC}, with the asymmetry scaled up now by a factor corresponding to $\pi/2$ and a further factor $(P_T \overline{P}_T)^{-1}$ ($\simeq$ 2.1), which yields an overall factor of ~ 3.3. The corresponding sensitivity is smaller is by the same factor. Indeed, the results we now obtain for $\mathrm{Im}\, v_{3,4}$ are related to those obtained from A_2^{CC} for $i = 2, 5$ by the same factor. For the case with longitudinal polarization, the sensitivities for the relevant $\mathrm{Im}\, v_i$ are enhanced by almost an order of magnitude, whereas the sensitivities for $\mathrm{Im}\, a_i$ are improved marginally.

In summary, from the asymmetry A_1^{CC}, we get $|\,\mathrm{Re}\, v_{3,4}| \leq 2.1 \times 10^{-4}$, $|\,\mathrm{Re}\, v_6| \leq 3.1 \times 10^{-3}$, and $|\,\mathrm{Re}\, a_{3,4}| \leq 3.1 \times 10^{-3}$, $|\,\mathrm{Re}\, a_6| \leq 4.6 \times 10^{-2}$, while the asymmetry A_2^{CC} yields the sensitivities $|\,\mathrm{Im}\, a_{3,4}| \leq 2.1 \times 10^{-4}$, $|\,\mathrm{Im}\, a_6| \leq 3.1 \times 10^{-3}$, and $|\,\mathrm{Im}\, v_{3,4}| \leq 3.1 \times 10^{-3}$, $|\,\mathrm{Im}\, v_6| \leq 4.6 \times 10^{-2}$. The asymmetry A_3^{CC} which can be defined for unpolarized and longitudinally polarized beams, yields the following sensitivities for unpolarized (longitudinally polarized) cases: $\mathrm{Im}\, v_{2,5} \leq 9.3 \times 10^{-4} (5.6 \times 10^{-5})$, $\mathrm{Im}\, v_6 \leq 1.4 \times 10^{-2} (8.4 \times 10^{-4})$ and $\mathrm{Im}\, a_{2,5} \leq 6.4 \times 10^{-5} (5.2 \times 10^{-5})$, $\mathrm{Im}\, a_6 \leq 9.6 \times 10^{-4} (7.9 \times 10^{-4})$.

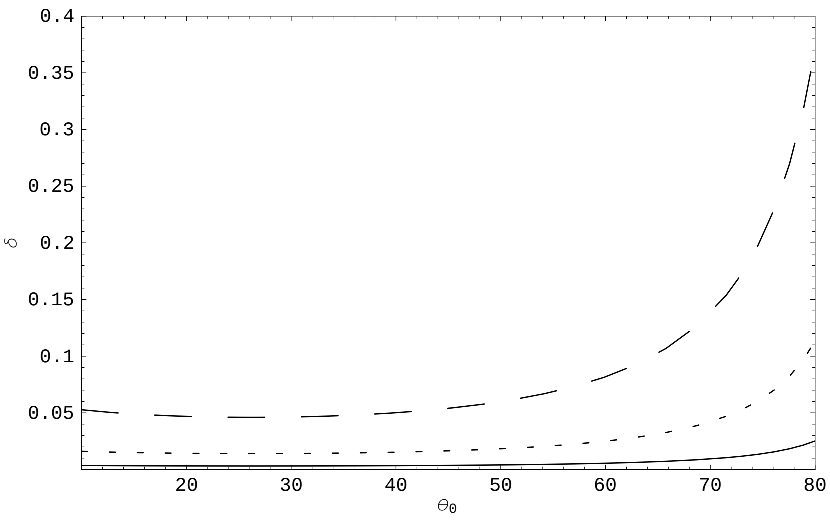

Figure 4: The 90% C.L. limit on Re v_6 from the asymmetry A_1^{CC} (solid line), and on Im v_6 from A_2^{CC} (dashed line) and A_3^{CC} (dotted line), plotted as functions of the cut-off θ_0.

It must be noted that the various couplings which are dimensionless arise from terms that are suppressed by different powers of m_Z^2. In particular, these must be viewed as model independent estimates, which could be used to constrain specific models.

4. Discussion and conclusions

It is now pertinent to ask what sort of models might lead to such CC and CV form-factors. In the context of the former, it was already shown in [8] that anomalous triple-gauge boson vertices[3] generate precisely the kind of correlations as a_6, v_6 provided we suitably identify the parameters. In the context of CV form-factors, Higgs models of the type considered in ref. [20] could give rise to couplings involving the ϵ symbol we have considered here. In ref. [21] CP even trilinear gauge boson vertices at one-loop in the SM and minimal supersymmetric model (MSSM) are computed, while the implications for colliders is considered in, e.g., ref. [22]. An earlier work discussing the implications of several different theoretical scenarios on the cross sections and angular distributions in $e^+e^- \to Z\gamma$ is [23].

In conclusion, we have considered in all generality the role of chirality conserving as well as chirality violating couplings due to physics beyond the standard model. The results due to the latter are entirely new and complement the former. We started out by elucidating the role of longitudinal as well as transverse polarization in phenomena such as these. By relating the number of independent helicity amplitudes to the possible number of CP violating (and conserving) amplitudes, we narrowed down the a linearly independent set of form-factors that could contribute to the differential cross-section. We have pointed out that for the case of chirality violation it is sufficient to consider only

[3] Here we do not give an exhaustive bibliography for the work done on this source of CP violation and refer instead to the references listed in a recent work [19].

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scalar type terms, as the tensor like terms are redundant, and is proof is provided in appendix A. We have constructed suitable asymmetries and have discussed their properties. These asymmetries have been employed to provide estimates for the level at which the new physics contributions may be constrained at the linear collider with realistic polarization and integrated luminosity. Our work provides a comprehensive set of expressions which can be used at future realistic detector environments and can be used to constrain and calibrate realistic detector signals.

Acknowledgments

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A. Redundancy of tensor interactions

Consider the massless Dirac equation,

$$\not{p}_1 u(p_1) = 0 \, ; \qquad \overline{v}(p_2)\not{p}_2 = 0 \, . \tag{A.1}$$

and the definition for the spin operator,

$$\vec{\Sigma} = \gamma_5 \gamma_0 \vec{\gamma} \, . \tag{A.2}$$

The following are then readily obtained:

$$\gamma_5 u(p_1) = (\vec{\Sigma} \cdot \hat{p}) u(p_1) \tag{A.3}$$

$$\overline{v}(p_2)\gamma_5 = \overline{v}(p_2)(\vec{\Sigma} \cdot \hat{p}) \tag{A.4}$$

where $\hat{p} \equiv \vec{p_1}/|\vec{p_1}| = -\vec{p_2}/|\vec{p_2}|$.

For a tensor matrix (i.e. anti-symmetrized product of two gamma matrices) T, then

$$\overline{v}(p_2)\gamma_5 T u(p_1) = \overline{v}(p_2)(\vec{\Sigma} \cdot \hat{p}) T u(p_1) \, , \tag{A.5}$$

$$\overline{v}(p_2) T \gamma_5 u(p_1) = \overline{v}(p_2) T (\vec{\Sigma} \cdot \hat{p}) u(p_1) \, . \tag{A.6}$$

Adding these two equations, and using the fact that γ_5 commutes with T, we now obtain

$$2\overline{v}(p_2)\gamma_5 T u(p_1) = \overline{v}(p_2)\{\vec{\Sigma}.\hat{p}, T\} u(p_1) \, . \tag{A.7}$$

We note here that the anti-commutator on the right hand side is an anti-commutator of two commutators of γ matrices. Well known identities involving such anti-commutators and commutators can be utilized to those that these reduce to combinations involving either 4 γ's (i.e. the product of the ϵ-symbol with γ_5) or no γ's. Thus the left hand side reduces to a combination of a pseudo-scalar and scalar. The exercise can be carried out by multiplying, at an earlier stage, by γ_5 yielding

$$2\overline{v}(p_2) T u(p_1) = \overline{v}(p_2)\gamma_5 \{\vec{\Sigma} \cdot \hat{p}, T\} u(p_1) \, , \tag{A.8}$$

and the same conclusion follows.

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B. CP properties of anomalous couplings

In order to establish the CP properties for the couplings in a transparent manner, it is useful to consider the effective lagrangian for the CC and the CV cases. These are chosen to be hermitean for the case that the couplings are purely real. These read as follows:

$$
\begin{aligned}
\mathcal{L}_I^{CC} = & \frac{e^2}{4\sin\theta_W\cos\theta_W m_Z^4}\cdot \\
& \cdot\Big\{ -i\left[\partial_\mu\bar\psi(V_1+A_1\gamma_5)\gamma_\beta\partial_\alpha\psi - \partial_\alpha\bar\psi(V_1+A_1\gamma_5)\gamma_\beta\partial_\mu\psi\right] - \\
& \quad -i\left[\bar\psi(V_2+A_2\gamma_5)\gamma_\alpha\partial_\beta\partial_\mu\psi - \partial_\beta\partial_\mu\bar\psi(V_2+A_2\gamma_5)\gamma_\alpha\psi\right] - \\
& \quad -i\left[\partial_\beta\bar\psi(V_3+A_3\gamma_5)\gamma_\alpha\partial_\mu\psi - \partial_\mu\bar\psi(V_3+A_3\gamma_5)\gamma_\alpha\partial_\beta\psi\right] + \\
& \quad +\left[\bar\psi(V_4+A_4\gamma_5)\gamma_\alpha\partial_\beta\partial_\mu\psi + \partial_\beta\partial_\mu\bar\psi(V_4+A_4\gamma_5)\gamma_\alpha\psi\right] + \\
& \quad +\left[\partial_\beta\bar\psi(V_5+A_5\gamma_5)\gamma_\alpha\partial_\mu\psi + \partial_\mu\bar\psi(V_5+A_5\gamma_5)\gamma_\alpha\partial_\beta\psi\right] - \\
& \quad - m_Z^2\bar\psi(V_6+A_6\gamma_5)\gamma_\alpha g_{\beta\mu}\psi\Big\} F^{\mu\alpha}Z^\beta\,,
\end{aligned}
\tag{B.1}
$$

and

$$
\begin{aligned}
\mathcal{L}_I^{CV} = & \frac{e^2}{4\sin\theta_W\cos\theta_W}\cdot \\
& \cdot\Big\{\frac{1}{m_Z^3}\bar\psi(S_1+iP_1\gamma_5)\psi F_{\mu\alpha}\partial^\mu Z^\alpha + \\
& + \frac{1}{m_Z^5}\left[\partial_\mu\bar\psi(S_2+iP_2\gamma_5)\partial_\alpha\partial_\beta\psi + \partial_\alpha\partial_\beta\bar\psi(S_2+iP_2\gamma_5)\partial_\mu\psi\right]F^{\mu\alpha}Z^\beta + \\
& + \frac{i}{m_Z^5}\left[\partial_\mu\bar\psi(S_3+iP_3\gamma_5)\partial_\alpha\partial_\beta\psi - \partial_\alpha\partial_\beta\bar\psi(S_3+iP_3\gamma_5)\partial_\mu\psi\right]F^{\mu\alpha}Z^\beta + \\
& + \frac{1}{2m_Z^3}\bar\psi(S_4+iP_4\gamma_5)\psi\epsilon_{\alpha\beta\rho\sigma}F^{\rho\alpha}\partial^\sigma Z^\beta - \\
& - \frac{i}{2m_Z^3}\left[\bar\psi(S_5+iP_5\gamma_5)\partial^\sigma\psi - \partial^\sigma\bar\psi(S_5+iP_5\gamma_5)\psi\right]\epsilon_{\alpha\beta\rho\sigma}F^{\rho\alpha}Z^\beta + \\
& + \frac{i}{m_Z^7}\left[\partial^\tau\partial_\mu\bar\psi(S_6+iP_6\gamma_5)\partial^\sigma\partial_\alpha\psi + \partial^\tau\partial_\alpha\bar\psi(S_6+iP_6\gamma_5)\partial^\sigma\partial_\mu\psi\right]\epsilon_{\beta\rho\sigma\tau}\partial^\mu F^{\rho\alpha}Z^\beta\Big\}.
\end{aligned}
\tag{B.2}
$$

The CP properties of various terms in the above lagrangians may be determined using the standard CP transformation properties of the electron, photon and Z fields. The result is that in $\mathcal{L}_I^{CC}$, the terms corresponding to $V_{1,2,3}$ and $A_{1,2,3}$ are CP even and the rest are CP odd. In $\mathcal{L}_I^{CV}$, the terms corresponding to $S_{1,2,5,6}$ and $P_{3,4}$ are CP even, while $S_{3,4}$ and $P_{1,2,5,6}$ are CP odd.

Once the CP properties are established from the effective lagrangians, it is no longer necessary to restrict the V_i, A_i, S_i, P_i, $i = 1, \ldots 6$ to be real, and we allow them to be complex. In accordance with the CPT theorem, the correlations arising from these will be CPT-even (odd) for the real (imaginary) parts of the form-factors.

Introducing plane-wave solutions for the e^+, e^-, Z and γ corresponding to the momenta given in eq. (3.1), and promoting the coupling constants V_i, A_i, $i = 1, \ldots, 6$ to complex

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form-factors dependent on kinematic invariants s, t and u, we find that to reproduce the vertex factor of eq. (3.4), we need to make the following replacements: $V_1 \to iv_1$, $V_2 \to i(v_2 - v_5), V_3 \to i(v_3 - v_4)$, $V_4 \to (v_2 + v_5)$, $V_5 \to (v_3 + v_4), V_6 = v_6$, and $A_1 \to ia_1$, $A_2 \to i(a_2 - a_5), A_3 \to i(a_3 - a_4)$, $A_4 \to (a_2 + a_5)$, $A_5 \to (a_3 + a_4), A_6 = a_6$. Analogously, for the CV case, again with S_i, P_i, $i = 1, \ldots, 6$, taken to be form-factors, we need to make the following replacements to reproduce eq. (3.5): $S_1 \to s_1$, $S_2 \to (s_2 - s_3)/2, S_3 \to -i(s_2 + s_3)/2$, $S_4 \to s_4$, $S_5 \to -is_5, S_6 \to -is_6$ and $P_1 \to p_1$, $P_2 \to (p_2 - p_3)/2, P_3 \to -i(p_2 + p_3)/2$, $P_4 \to p_4$, $P_5 \to -ip_5, P_6 \to -ip_6$. The CP properties of the momentum-space form-factors, which are stated explicitly in the text and tabulated in tables 1 and 2 may be easily deduced from the respective CP properties of the couplings in the lagrangian.

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